

Comparative Study of Muqarnas with Takht and Brick-Facing Elements at Isfahan's Historical Mosques Through a Two-Dimensional Geometry of Muqarnas*

Elham Rezaei^a - Shahryar Nasekhian^{b**} - Nima Vali-Beig^c

^a Ph.D. Candidate in Restoration of Historical Buildings and Textures, Faculty of Preservation and Restoration, Isfahan University of Art, Isfahan, Iran.

^b Associate Professor of Restoration of Historical Buildings and Textures, Faculty of Preservation and Restoration, Isfahan University of Art, Isfahan, Iran (Corresponding Author).

^c Assistant Professor of Restoration of Historical Buildings and Textures, Faculty of Preservation and Restoration, Isfahan University of Art, Isfahan, Iran.

Received 18 September 2023; Revised 02 August 2024; Accepted 27 August 2024; Available Online 20 December 2024

ABSTRACT

A two-dimensional geometry framework helps investigate the similarities and differences of muqarnas¹ with Takht and brick-facing features at Isfahan's historical mosques. Analyzing similarities and differences will thus reveal the specific and general features of the said muqarnas. A comparative study technique is a strategy to investigate how and why muqarnas are similar and what makes them distinguish from each other. No comparative studies have ever employed a two-dimensional geometric design to understand and categorize muqarnas with Takht and brick-facing compositions at the historical mosques of Isfahan, Iran. The statistical population of this study was composed of eight muqarnas at Isfahan's historical mosques, including the Lonban Mosque, the Hakim Mosque, the Jame' Mosque of Isfahan, the Saro-Taghi Mosque, the Ilchi Mosque, the Shishe Mosque, and the Sha'ya Mosque, where muqarnas formations are constructed in Nimkar shapes. Data were collected by acquiring the muqarnas using short-range photogrammetry and point cloud modeling, as well as by taking plane photos of the models before plotting them in AutoCAD. Meanwhile, geometric and mathematical relations were used to analyze the data. Disregard the differences, there are many similarities with the muqarnas when it comes to examining the interrelationships between elements of the plan, proportions, and divisions of the plan, and facing features, with the Tee² element being missing in all the muqarnas sites. Repetitive features between muqarnas provide solutions to plot them into two-dimensional forms. The most important solutions include the proportions of the main Takhts to the side length of Toranjs, the circumscribed or inscribed circles of Shamse or the circumscribed circle of the Toranjs on the midpoints of the length, width, or diameter of half of the context, making the side length of the Toranj equivalent to half the height of tiers, and also making large sides of Shaparaks and Pabariks approximately equivalent to the Toranj's side.

Keywords: Brick-Faced Muqarnas, Takht Element, Two-Dimensional Geometry, Isfahan's Historic Mosques.

* This article is derived from the first author's Ph.D. dissertation entitled "Providing a Geometric Algorithm for Recovering Missing Parts of Muqarnas with Brick Faces and Takht Elements in Plate Geometry; Case Study: Muqarnas of Historical Mosques in Central Iran". This article was read by the second and third authors at Isfahan University of Art.

** E_mail: s.nasekhian@aui.ac.ir

1. INTRODUCTION

Muqarnas (also, stalactite vaulting) was an additional part of the building and was first used in the corners of domes to cover up the empty distances between the main body of the building and the dome. Over time, this part used to serve as a secondary shell in the coverings of porches (iwans), portals, and undersides of the niches or domes, and was predominately used as an ornamental component. Muqarnas falls under different types and categories based on the presence of the element³ of Takht⁴ and the materials used to construct them. Muqarnas is classified into two categories based on how it is embedded in a position. The first category of muqarnas includes one that surrounds a volume constructed such as a capital or minaret⁵ muqarnas, and the second one includes an enclosed muqarnas surrounded by a wall on two or more sides. There is also another type of muqarnas called Nimkar⁶ or Cheshme muqarnas. Muqarnas can be defined in different respects, as suggested by different sources. Muqarnas is said to involve some horizontal rows of equal distances composed of muqarnas elements. The elements, linking the lower elements to the upper elements, help create fill and empty spaces to generate a tier. A set of tiers and a central Shamse would constitute a muqarnas. By a muqarnas with a brick facing, it is meant one whose main material on its facing is brick and it is indeed constructed out of brick.

This study aimed to investigate the historical muqarnas featuring brick facing and Takht elements. Despite their fewer samples elsewhere in the world, muqarnas of this kind are abound in Isfahan and in its historical mosques and schools. For this, the study consisted of Isfahan's historical mosques as its target population, with most of the muqarnas constructed in the form of Nimkar or squinch.

A two-dimensional geometry design incorporated into brick geometric features helps categorize the properties of muqarnas with brick facing and Takht elements while revealing their similarities and distinctions, especially in Isfahan's historical mosques. Analyzing similarities and differences would reveal the specific and general features of the said muqarnas at Isfahan's historical mosques. The core questions of the study are as follows: 1- What similarities and differences are revealed by the two-dimensional geometry in the muqarnas with brick facing and Takht elements at Isfahan's historical mosques? 2- If similarities are assumed to outweigh differences, which factors cause the differences to arise? 3- What compositions are represented by the repetitive arrangements around the Takht elements? And 4- Which solutions are provided by the repetitive features used in analyzing the plane geometry of the muqarnas for plotting the plan and the muqarnas facing?

Previous research has paid less focus on brick-constructed muqarnas with the Takht element, failing

to investigate details of plotting, proportions, and repetitive compositions in these types of muqarnas, despite circular brick features.

2. LITERATURE REVIEW

Research on the two-dimensional geometry of muqarnas falls under several categories: one category views muqarnas from a history of mathematics perspective using the views of Ghias al-Din Jamshid Kashani, who primarily analyzed plans. This category of research is concerned with calculating the surface areas of the muqarnas, types of arches between the tiers, and polygons constituting planes (Özdural 1990; Özdural 1991; Dold-Samplonius 1992; Zamani, Fattahi, and Ekhtiari 2019). The research has demonstrated that Kashani (2008) analyzed only one special type of muqarnas, demonstrating that plan elements in this specific muqarnas were mainly octagonal. The muqarnas, as considered by previous research, lack the plurality of muqarnas elements, including the Takht element.

Another category of research is concerned with the evolving muqarnas plans, their models, and how they were plotted. The book "The Topkapi Scroll—Geometry and Ornament in Islamic Architecture" introduces the first muqarnas-shaped plan, the Takht-e-Sulaiman Plaster Tablet, and Tashkent and Topkapi scrolls. This book demonstrated that by the late 15th century, a radial secondary pattern grid had formed in the muqarnas plan, replacing the previous-era perpendicular grids (Necipoğlu 2018). While focusing on the two-dimensional radial secondary patterns in the Topkapi Scroll, this book does not offer a technique to plot muqarnas; rather it modifies the geometry of the plans in many respects while disregarding the differences as to how the plans utilize geometric principles. In the studies "The Use of Muqarnas in the Transitional Zone of Domes in Egyptian Islamic Architecture" and "Bahri Mamluk Muqarnas Portals in Egypt: Survey and Analysis", Fukami (2017) and Kashef (2017) respectively investigated evolving plans during the Egyptian muqarnas history. In another study, Büyükdığın (2001) investigated the geometric order of dome plans with stucco muqarnas in Turkey's Edirne. These three studies have failed to investigate the element of Takht in muqarnas compositions, and do not reveal the plurality and diversity of other elements in the respective plans, either. Meanwhile, these researchers offer explanations about the rules of plotting the plan and constructing muqarnas, maps of different types of muqarnas, rules of specifying the edges of the tiers and introducing their elements (Reiszade and Mofid 2014; Lorzade 1980).

While the research deals with muqarnas's two-dimensional geometries, they consider the main user as an individual who constructs a muqarnas rather than provides a design, thereby failing to prioritize

details of how a muqarnas plan is plotted. Concerning "Gereh⁷ and Karbandi", the above research identifies constituting elements of muqarnas, defining how they are plotted on a two-dimensional plane by citing three samples of two-, four-, and six-tier muqarnas (Sharbaf 2006). However, despite focusing on muqarnas featuring the element of Takht, the research has failed to provide any explanations on details of plotting the plans and their sizes or dimensions. In the book "Practical Training of Muqarnas", plan plotting and two-dimensional geometry have been examined. Piran (2018) details how a muqarnas is plotted in circular, rectangular, oval, and polygonal contexts. This book has provided no such a fixed design under various contexts, demonstrating certain (non-integral numbers) sizes as radii for the arcs, which are not clear how they were calculated.

The other category of research, "Interrelation between the Perpendicular Grid Systems and Star Polygons of Muqarnas Ground Projection Plans" and "Origin of Irregular Star Polygons in Ground Projection Plans of Muqarnas", Agribas et al. (2022) have demonstrated that Turkey's stone-made muqarnas have applied oval grids or a mix of spiderweb grid and a square grid for plan composition (Agribas and Yildiz 2021; Agribas, Yildiz, and Sahin, 2022). This research just analyzed a certain type of muqarnas, which differs from the muqarnas in the present study not only in terms of consecution materials but also in terms of the plan. More research was concerned with how muqarnas could be classified based on their plan metrics. On an Internet Database, Takahashi collected and shared samples of muqarnas, demonstrating three different styles of muqarnas based on plan designs. Meanwhile, Takahashi failed to provide a classification of highly diverse radius-styled muqarnas, while describing muqarnas with octagonal elements under a square category. In his doctoral dissertation, "Muqarnas Geometry in Islamic Architecture", Sakkal (1982) classified muqarnas domes of regular blocks based on their plans. Another study by Kilicoglu (2022) "An Essay on the Classification of Muqarnas: Geometric Foundations Based on Application in Practice" classified muqarnas into three categories where the elements constituting the muqarnas plan were generated through a regular octagon, with the angles of the elements being an integral multiple of 22.5°, or through a regular hexagon, with the angles of the elements being an integral multiple of 30°, or through a regular decagon, with a base angle of 18°. The classifications provided by these two studies also involved a limited number of muqarnas, excluding those that feature Takht and a diversity of angles in their plans.

The last category of research utilizes cutting-edge technologies to analyze muqarnas plans. In his doctoral dissertation, "Algorithmic Computer Reconstructions of Stalactite Vaults- Muqarnas -in Islamic Architecture", Silvia Harmsen (2006) employed the Graph Theory to provide all three-dimensional information of the Seljuk and Ilkhanate-era muqarnas in the plan. This study deals with how to reconstruct the said muqarnas when collapsed or by assuming that respective plans are available (Harmsen 2006). The muqarnas under consideration in his dissertation correspond to the definition by Ghias al-Din Jamshid Kashani based on a regular octagon. Güzelci et al. (2021) also provided a novel computational model to measure the entropy value of two-dimensional geometric models in the study "Measuring the Entropy of Sinan's Muqarnas Patterns". In the study "The Study on Measuring Complexity in Muqarnas Patterns", Güzelci and Alaçam (2019) also measured muqarnas plan complexity and evaluated the complexity of a sample muqarnas plan, which was an octagonally based muqarnas with square and oval elements.

Past research conducted on muqarnas with two-dimensional geometric features has failed to specifically deal with brick muqarnas containing the element of Takht and the muqarnas containing diverse plan elements. While stucco and tile-worked muqarnas plans with the element of Takht involve violations of geometric features, brick muqarnas seldom expose any violation of geometric features due to their modular properties, construction materials, and dimension limitations. From this perspective, a two-dimensional geometry may well serve to compare these types of muqarnas. In sum, this study aimed to describe in detail muqarnas plans by examining the specific and general features of different muqarnas.

3. THEORETICAL FOUNDATIONS

The present study discusses prominent theorists who have examined muqarnas based on two-dimensional geometries. Dold-Samplonius approached muqarnas designs from the perspective of the history of mathematics by focusing on Kashani's views; Yaghan investigated muqarnas from the viewpoint of the evolutionary understanding of the West Islamic world (with plans consisting of squares and rhombs); Harmsen used mathematics and the Graph Theory to examine muqarnas of the Seljuk and Ilkhanate eras; Güzelci and Alaçam examined muqarnas through digital designs, while Hossein Lorzade and Asghar Sharbaf focused on the muqarnas configuration and geometry plotting, respectively (Table 1).

Table 1: Prominent Theorists involved in Muqarnas Geometry

Prominent Theorists	Approaches	Description
Dold-Samplonius	History of Mathematics (Muqarnas Surface Calculation)	Following Kashani, Dold-Samplonius (1992) defines muqarnas to be composed of cells and middle elements and analyzes the Kashani technique in calculating the muqarnas surface. His computations of the area of the middle elements and cells in curved muqarnas were found to be lower than those of Kashani in two cases while being higher in two other cases. He claims Kashani fixed the values.
Jalal Yaghan	Evolutionary View of the West Islamic World (with a Focus on the Plan)	Using an evolutionary view of the West Islamic world, Yaghan (2001) presents two new classes of pre-designed erecting units (Fig. 1). He generates some innovative forms to link with the past but not repeat it. The plan elements of this kind of muqarnas are either squared or rhombic, which he divides into smaller triangles, thereby seeking to form newer units based on triangles.
Silvia Harmsen	Employing the Graph Theory in the Plan	Harmsen (2006) employed the Graph Theory to solve the problem of recognizing the edge of the tiers in the muqarnas attributed to the Seljuk/Ilkhanate eras, arguing that the theory can help represent all the three-dimensional information of the muqarnas in the plan. His theory could help reconstruct this group of muqarnas three-dimensionally in the event that the plan is available.
Güzelci and Alaçam Examined	Digital Design Approach	Güzelci et al (2021) maintain that the digital design approach helps explore undiscovered muqarnas potentials while helping calculate the plan entropy. The outcomes of these calculations could be used to understand stylistic characteristics in the history of architecture.
Hossein Lorzade	Geometry of Plotting and Constructing Muqarnas	Matching of the muqarnas facing with the Nimkar arcs in the Nimkar muqarnas: Lorzade (1980) divides muqarnas into three categories based on materials. The muqarnas that are constructed simply by stucco or mirror covering; the muqarnas that are constructed out of brick, and the muqarnas that are covered by tiles and tilework. He calls the elements constituting muqarnas Aalat (instrument), defining Tas ⁸ , Takht, and Madani ⁹ as the main instruments, conjoined with the instruments of Toranj ¹⁰ and Shaparak ¹¹ . He also calls the instrument of the tier footing Moushpa ¹² . To him, Moushpa can be elongated or shortened at any rate, but it must not be shorter than the height (Lorzade 1980).
Asghar Sharbaf	Geometry of Plotting and Constructing Muqarnas	Asghar Sharbaf (2006) uses the term Karbandi ¹³ for all three-dimensional patterns (muqarnas, Rasmibandi ¹⁴ , Kase-Sazi, and Yazdi-Bandi), defining muqarnas as a subset of Karbandi. He also calls the elements constituting muqarnas Aalat, classifying muqarnas into two headings Takht and non-Takht Aalats. He emphasizes that muqarnas should be laid on the radius of the Shamse lines, suggesting that plotting should begin from where the Takhts are positioned.

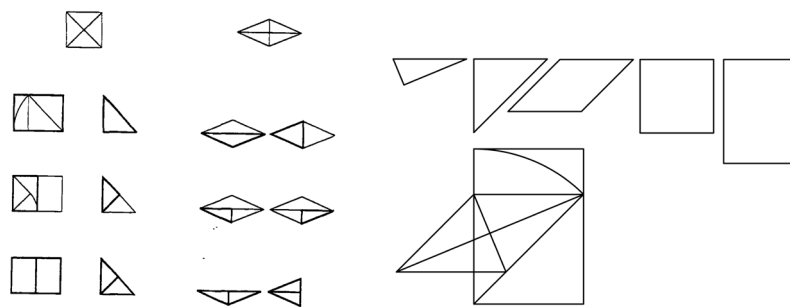


Fig. 1. West Islamic World Muqarnas Elements (from Left: Two Classes of New Pre-Designed Units; from Right: The Main Units Constituting Square-Based Muqarnas)
(Derived from Yaghan 2001)

As given in Table 1, the study relies on the views of Hossein Lorzade and Asghar Sharbaf who specifically focused on the geometry of plotting and constructing muqarnas. These were the only theorists who took into account muqarnas with Takht elements.

4. DESCRIPTION AND EXAMINATION

This section is concerned with the methodology and discussion of the study.

4.1. Methodology

The methodology involves how data are collected and analyzed. This study used the field survey and the library study to collect data. The field survey involves manual acquisition, imaging, and using short-range photogrammetry techniques, including the three-dimensional point cloud modeling of the muqarnas, and extracting the plane photo of the point cloud model, followed by plotting it in the AutoCAD environment. The point cloud modeling and the plane photo were carried out in the Agisoft Metashape (Fig. 2). Being quantitative in nature, this study used mathematical and geometric relations to analyze the data. To simplify the analyses, the muqarnas elements and their names were assigned abbreviated signs and colors (Table 2). Muqarnas of the mosques under study were selected based on the latest registered list of Isfahan's historical mosques. The muqarnas included the squinch muqarnas of the Aghanour Mosque (the Safavid era), the squinch of the Hakim Mosque, the

Nimkar of the Hakim Mosque's mihrab¹⁵, the Nimkar of the Hakim Mosque's eastern portal (the Safavid era), the Nimkar of the Jame' Mosque of Isfahan's apprentice platform (from the Sejuk to Qajar era), the Nimkar of the Saro-Taghi Mosque's entrance (the Safavid era), the Nimkar of the Shishe Mosque's entrance (the Safavid era), the Nimkar muqarnas of the Sha'ya Mosque's entrance (the Safavid era), the Nimkar of the Ilchi Mosque's mihrab¹⁶ (the Safavid era), and the Nimkar of the Lonban Mosque (the Safavid era).

The historical eras of the mosques are based on registered case information. Considering the frequency of the Nimkars and the presence of two muqarnas squinches among the samples, the Aghanour Mosque's muqarnas squinch and the Hakim Mosque's squinch were removed from the statistical population in order to obtain the features under consideration from an identical context.

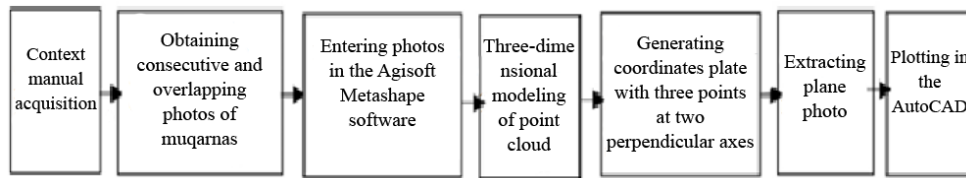























Fig. 2. Digital Documentation Process of Muqarnas

Table 2: Abbreviated Signs, Colors, and Names assigned to each of the Muqarnas Elements

Name of the Element from the View of Theorists and Authors			Abbreviated Signs	Muqarnas Element Plan	Muqarnas Element Plan
Lorzade	Sharbaf	Authors			
Shamse	Shamse	Shamse	 S		
Toranj	Toranj	Toranj	 R		
Tas (Taseh)	Tas	Tas	 T		
Shaparak (Parak)	Shaparak	Shaparak	 H		
Neizeh	Shaparak	Parak ¹⁷	 P		
Toranj	Pabarik ¹⁸	Pabarik	 B		
Madani	Tanoureh	Madani	 M		

Name of the Element from the View of Theorists and Authors			Abbreviated Signs	Muqarnas Element Plan	Muqarnas Element Plan
Lorzade	Sharbatf	Authors			
Susan	Sarseft ¹⁹	Sarseft	■ F		
Takht (if it involves over 6 points), it would be a Shamse)	Multi-Pointed Takht	Takht	□ A		-
Tee ²⁰	-	Tee	■ I		

4.2. Analysis and Discussion

Two-dimensional geometry reveals the similarities and differences of muqarnas at Isfahan’s historical mosques within three main subsets 1: interrelations between plan elements, 2: plan proportions, and 3: facing element features

4.2.1. Interrelations between Plan Elements

The interrelationship between plan elements is concerned with evaluating two issues: geometric features²¹ and the interrelationship between elements within a tier (Table 3). This interrelationship is only examined in half of the plan due to the reflective symmetry of the plan’s centerline in all muqarnas.

Table 3: Interrelationship between Muqarnas Plan Elements

Name	Geometric Features	Interrelationship between Elements within a Tier (in Half of the Plan)
Nimkar of the Lonban Mosque	 Two Reflective Symmetries	<p>Tier 3 TR,M,R,T,R,M,R,T</p> <p>Tier 2 FP,TP,M,P,B,T,B,T,B,T,B,P,M,P,B,T,B,F</p> <p>Tier 1 HB,P,PH,F,H,H,T,H,H,F,H,P,B,P</p>
Nimkar of the Hakim Mosque’s Mihrab	 Two Reflective Symmetries	<p>Tier 4 T,R,M,R,T,R,M,R,T</p> <p>Tier 3 T,P,T,H,M,H,B,T,B,T,B,T</p> <p>Tier 2 P,T,H,T,P,H,T,H,F,H,T,H,F,H,T,H,P,T,H,T,P</p> <p>Tier 1 H,B,H,H,B,H</p>
Nimkar of the Hakim Mosque’s Eastern Entrance	 Two Reflective Symmetries	<p>Tier 4 T,R,M,R,T,R,M,R,T,R,M,R,T</p> <p>Tier 3 T,P,T,H,M,P,B,T,B,T,B,P,M,P,B,T,B,T,B,P,M,P,B,T,B</p> <p>Tier 2 T,H,H,B,T,B,H,T,H,B,T,B,H,T,H,B,T,B,H,T</p> <p>Tier 1 P,H,F,P,P,T,H,P,P,H,T,P,P,F,H,P</p>
Nimkar of the Saro-Taghi Mosque’s Entrance	 One Reflective Symmetry	<p>Tier 3 T,R,T,R,T,R,T</p> <p>Tier 2 T,H,T,H,B,T,B,T,B,T,B,H,T,H,F</p> <p>Tier 1 H,H,H,H,H</p>

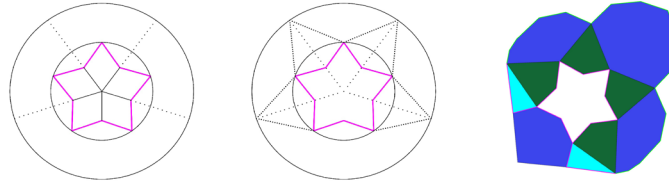


Fig. 3. Tanoureh with a Five-Pointed Takht (from Left: How Pabariks are Plotted; from Right: Highly Repetitive Tanoureh from the Muqarnas in the Table)

Examining all Tanoures in the statistical population of muqarnas reveals that in all the Tanoures, the Tase lies between the Pabariks conjoined to the Takht, with only one exception between the two samples. In the Shishe Mosque's muqarnas, a Madani stands between

two Tanoureh Pabariks, while there is a Sarseft lying between two Pabariks in the eastern porch muqarnas of the Jame' Mosque of Isfahan. The arrangement around the non-Tanoureh Takhts in the samples under research is usually as follows (Fig. 4):

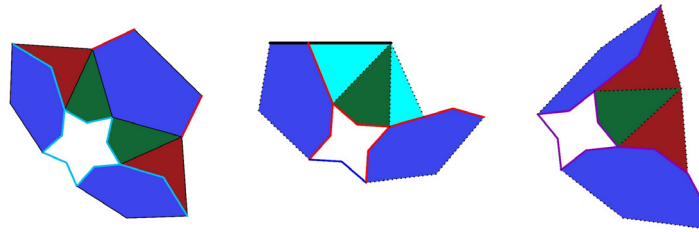


Fig. 4. The Arrangement Around the Non-Tanoureh Takhts; Sum of Takht, Two Tases on Either Side and the Pabarik



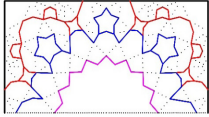

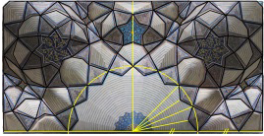
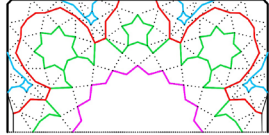
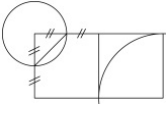
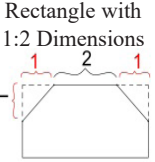
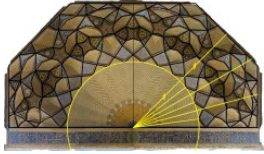
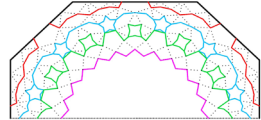
In non-Tanoureh Takhts, both sides of the Takht are conjoined with the Tase from either axis of the Takht symmetry, with the remaining sides of the Takht conjoined to the Pabarik. The Shaparak or Parak is conjoined with the Pabarik and shares a side with it.

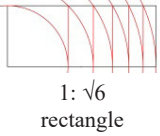
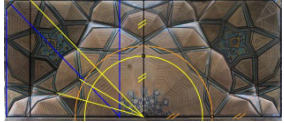
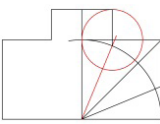
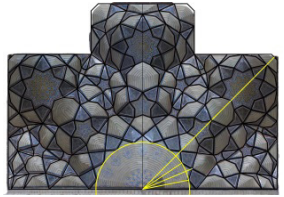
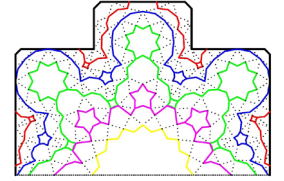

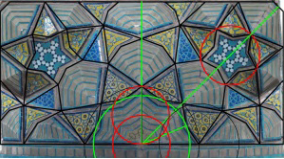
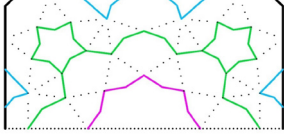
4.2.2. Proportions and Plan Divisions

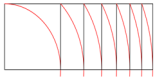
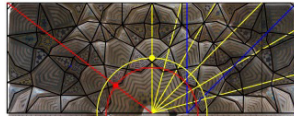
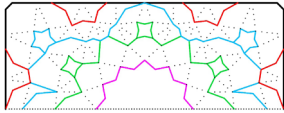
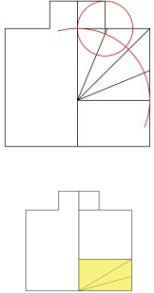
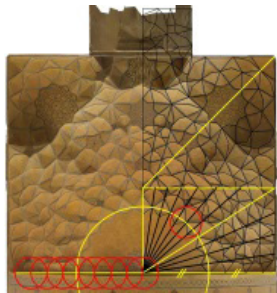
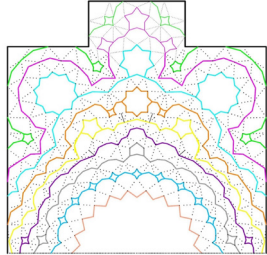
This section concerns the type of context and proportions of Shamse, Toranj, Shaparak, Pabarik,

and Takht relative to the context in each muqarnas (Table 4). While there is a thin Nimkar in the samples, in most muqarnas under study, the context is a square-shaped Nimkar or is generated through a square-shaped Nimkar. The square Nimkar is defined to be a rectangle consisting of two conjoining squares, i.e., a rectangle whose length is twice as much as its width (Reiszade and Mofid 2014, 38).

Table 4: Muqarnas Plan Proportions

Type of Muqarnas	Context	Type of Shamse/Angle Unit	The Circumscribed Circle of the Shamse	The Inscribed Circle of the Shamse	The Circumscribed Circle of the Toranj's	Toranjs' Length of the Sides ²³	Takht Proportions	Takht and Plan Images
Nimkar of the Lonban Mosque (Three Tiers)	Rectangle with 1:2 Dimensions 	Shamse 16, 11.25°	With a radius of 1/2 of the side of the square (half the context)	---	---	1/6 of the square side (half the context)	<ol style="list-style-type: none"> Five-pointed Takht: The side length of the circumscribed pentagon of this Takht equals that of the Toranj. Four-pointed Takht: the radius of the circumscribed circle of this Takht is 1/3 of the diameter of the circumscribed circle of the five-pointed Takht. 	  Plan
Nimkar of the Hakim Mosque's Mihrab (Four Tiers)	Rectangle with 1:2 Dimensions 	Shamse 16, 11.25°	With a radius of 1/2 of the side of the square (half the context)	---	---	1/6 of the square side (half the context)	<ol style="list-style-type: none"> Seven-pointed Takht: The radius of the circumscribed circle of the Takht equals the side of the Toranj. Five-pointed Takht: The side length of the circumscribed pentagon of this Takht equals that of the Toranj. Four-pointed Takht: The diameter of its circumscribed circle is 1/5 of the diameter of the five-pointed star. 	 The circumscribed circle of the Shamse  Plan
Nimkar of Hakim Mosque's Eastern Entrance (Four-Tiers)	Irregular Half Octagon  Rectangle with 1:2 Dimensions 	Shamse 24, 7.5°	---	With a radius of 1/2 of the diagonal angle segment	With a radius of 2/3 of the side of the half the context	1/9 of the length of half the context	<ol style="list-style-type: none"> Four-pointed Takht: The side length of the circumscribed quadrilateral of this Takht equals that of the Toranj. Five-pointed Takht: The radius of the circumscribed circle of the five-pointed Takht equals that of the four-pointed Takht. 	 The Inscribed circle of the Shamse  Plan

Type of Muqarnas	Context	Type of Shamse/Angle Unit	The Circumscribed Circle of the Shamse	The Inscribed Circle of the Shamse	The Circumscribed Circle of the Toranjs	Toranjs' Length of the Sides ²³	Takht Proportions	Takht and Plan Images
Nimkar of Saro-Taghi Mosque's Entrance (Three Tiers)	Elongated Rectangle (thin Nimkar)  1: √6 rectangle	Shamse 12, 15°	---	Approximately with a radius of 1/2 of the length of half the context	With a radius of 2/3 of the side of the half the context	1/5 of the length of half the context	Five-pointed Takht: The side length of its circumscribed pentagon equals that of the Toranj. This design features two main diagonal axes; one for Shamse and the other for the Takht.	 Inscribed circle of the Shamse between the two circles
Nimkar of Sha'ya Mosque's Entrance (Five-Tiers)	Combination of two 1: 2 Rectangles (Nimkar) 	Shamse 16, 11, 25°	---	With a radius of 1/4 the diameter of the large square of the context	With a radius of 1/2 of the length of the half the context	1/8 of the square side (half the context)	Large five-pointed Takht: The side length of its circumscribed pentagon equals that of the Toranj. Large eight-pointed Takht: The side length of its circumscribed octagon equals that of the Toranj. Large four-pointed Takht: The side length of its circumscribed quadrilateral equals 1/2 of that of the Toranj. Small five-pointed Takht: The side length of its circumscribed pentagon equals 1/2 of the side of the Toranj. Small eight-pointed Takht: (on a vertical axis): Its circumscribed circle radius is smaller than that with a radius of 0.1 of the large eight-pointed Takht's circumscribed circle.	 Inscribed circle with a radius of 1/4 of the diameter  Plan
Nimkar of Ilchi Mosque's Mihrab (Three Tiers)	Rectangle with 1:2 Dimensions 	Shamse 8, 22.5°	---	With a radius of 1/4 the diameter of the large square of the context	A little less than 2/3 of the square side of the context	2/7 (a little over 1/4) of the length of half the context	Five-pointed Takht: The circumscribed circle radius of this Takht equals half of the Shamse's circumscribed circle radius The side length of the Takht's circumscribed pentagon is smaller than that of the Toranj.	  Plan

Type of Muqarnas	Context	Takht Proportions	Takht and Plan Images
Nimkar of Shishe Mosque's Entrance (Four-Tiers)	<p>Elongated Rectangle (Thin Nimkar)</p>  <p>1: $\sqrt{7}$ Rectangle</p> <p>Shamse 12, 15°</p> <p>Between two circles with a radius of 1/2 the context's width and of 1/4 of the diameter of the half the context</p>	<p>1. Four-pointed Takht: The diameter of the circumscribed quadrilateral of this Takht equals the side of the Toranj</p> <p>2. Five-pointed Takht: The diameter of the circumscribed pentagon of this Takht equals the side of the Toranj</p> <p>This design has two main diagonal axes; one for the Shamse and the other for the Takht.</p>	 <p>Divisions and Circumscribed Circle of the Shamse</p>  <p>Plan</p>
Nimkar of Jame' Mosque of Isfahan's Apprentice Platform (Nine Tiers)	<p>Combination of two 1: 2 Rectangles and an Elongated Rectangle (Nimkar)</p>  <p>Shamse 22, 8, 18°</p> <p>1/2 the length of half the context</p>	<p>1. Round-the-Shamse small four-pointed Takht: Its circumscribed circle radius equals 1/3 of the Toranj's side,</p> <p>2. Ten-pointed Takht: Its circumscribed circle diameter equals 3 times the Toranj's side</p> <p>3. Five-pointed Takht: Its circumscribed circle diameter equals 1/2 of the circumscribed circle diameter of the ten-pointed Takht.</p> <p>4. Eight-pointed Takht: Its circumscribed circle radius equals the Toranj's side.</p> <p>5. Seven-pointed Takht: The side length of its circumscribed heptagon equals that of the Toranj.</p>	 <p>Propositions of Toranj and the Circumscribed Circle of the Shamse</p>  <p>Plan</p>

4.2.3. Muqarnas Features on the Facing

This section deals with the type of the facing arch, arch proportions between the tiers, and the relationship

between the tier height and the length of the Toranj's side (Table 5).

Table 5: Muqarnas Features on the Facing

Features	Muqarnas			
	Type of Facing Arch		Arch Proportions between the Tiers	Relationship between Tier Height and the Side Length of the Toranj
Nimkar of the Lonban Mosque (Three-Tier Muqarnas)	Sharp Panj/Haft		2/3 vertical section 1/3 curved section	Toranj's side length = half the tier's height
Nimkar of the Hakim Mosque's Mihrab (Four-Tier Muqarnas)	Simple Patupa		2/3 vertical section 1/3 curved section	Toranj's side length = half the tier's height
Nimkar of the Hakim Mosque's Eastern Entrance (Four-Tier Muqarnas)	Sharp Panj/Haft		2/3 vertical section 1/3 curved section	Toranj's side length half the tier's height
Nimkar of the Saro-Taghi Mosque's Entrance (Three-Tier Muqarnas)	Sharp Panj/Haft		2/3 vertical section 1/3 curved section	Half the tier's height < Toranj's side length Toranj's side = half the tier 1/5+half the tier
Nimar of the Sha'ya Mosque's Entrance (Five-Tier Muqarnas)	Sharp Panj/Haft		2/3 vertical section 1/3 curved section	Toranj's side length half the tier's height
Nimkar of the Ilchi Mosque's Mihrab (Three-Tier Muqarnas)	Sharp Panj/Haft		1/2 vertical section 1/2 curved section	Half the tier's height < Toranj's side length Toranj's side = half the tier 2/3+half the tier
Nimkar of the Shishe Mosque's Entrance (Four-Tier Muqarnas)	Sharp Panj/Haft		2/3 vertical section 1/3 curved section	Half the tier's height < Toranj's side length Toranj's side = half the tier 2/5+half the tier
Nimkar of the Jame' Mosque of Isfahan's Eastern Iwan (Nine-Tier Muqarnas)	Slow Shabdari		1/2 vertical section 1/2 curved section	Tier's height = Toranj's side length

All tiers feature an equal height except for the first tier. Since the Toranj is generated based on the Shamse's angular lines and is exactly based on plane geometry, the proportions of arch heights between tiers in each muqarnas are measured based on its side length. In half of the samples, the Toranj's side length is half the

tier's height.

4.3. Findings

According to the above-mentioned material, the study's findings are as follows (Table 6):

Table 6. Findings

	Interrelations between Plan Elements	Proportions and Divisions of Plan Elements	Muqarnas Facing Features	
General Features of Muqarnas	In all Muqarnas	<ul style="list-style-type: none"> - They lack a Tee element - All muqarnas lack a Sarseft in Tiers 3 onwards 	<ul style="list-style-type: none"> - The more Shamse points, the smaller Toranj's side size to the context will become and vice versa; - Main Takht proportions (on the axis of angles) are associated with the side length of the Toranj - Takht 5 is the most common in muqarnas 	-
	In most Muqarnas	<ul style="list-style-type: none"> - There is a discontinuity in the first tier of all muqarnas except for the Lonban Nimkar. - There is a continuity in the second tier of all muqarnas, suggesting that all adjacent elements share a side. The only exception is the muqarnas Nimkar of the entrance to the Sha'ya Mosque whose second tier features a discontinuity between the elements. - Five muqarnas out of eight feature a Madani in the last tier, i.e., the Madani's top is at the last tier. - Most muqarnas often feature the combination of conjoined Shaparaks in the first tier. - In seven muqarnas, the trails of the Tas, Pabarik, and Tas are highly repetitive in the tier before the last one. 	<ul style="list-style-type: none"> - In seven muqarnas, the 45° angle between all Shamse angles is the lowest common multiple. - In seven muqarnas, the midpoint of the length, width, and diameter of half of the context is located in the Shamse's circumscribed or inscribed circle or in the Toranj's circumscribed circle. - In six muqarnas, there is a Takht whose circumscribed polygonal side length equals the Toranj's side. 	<ul style="list-style-type: none"> - In six muqarnas, the arch between the tiers features 2/3 of the vertical section and 1/3 of the curved section. - In six samples, the Nimkar arch is one of sharp Panj/Haft
Particular Features of Muqarnas	In Few Muqarnas	<ul style="list-style-type: none"> - Three muqarnas in the first tier feature a Sarseft. - Three muqarnas in the second tier feature a Sarseft. 	<ul style="list-style-type: none"> - In the Shishe Mosque's muqarnas and the Ilchi Mihrab, there is no Takht whose circumscribed polygonal side length equals the side of the Toranj. - In two muqarnas of the Sha'ya mosque and Apprentice Platform, there is a Takht with over seven points. 	On the eastern iwan of the Jame' Mosque of Isfahan and at the Ilchi mihrab, the arch between the tiers features 1/2 of the vertical section and 1/2 of the curved section.
	In a Special Case	<ul style="list-style-type: none"> - Only in the first-tier muqarnas of the Nimkar of the Lonban Mosque do the adjacent elements share a side with each other, with no discontinuity between them. - The Shishe Mosque's muqarnas features a Madani in another tier except for the last one (the Madani's top is in another tier except for the last one) - In the penultimate tier, the Sarseft is adjacent to the Shaparak only in the Saro-Taghi muqarnas. 	<ul style="list-style-type: none"> - In the apprentice platform muqarnas, an angle of 45° is not multiple of the Shamse angles. - In the Ilchi muqarnas, the midpoint of the length, width, and diameter of half of the context is neither located in the Shamse's circumscribed circle nor in the inscribed circle of the Toranj. 	-

5. CONCLUSION

Findings showed that all eight muqarnas under study lacked the element of Tee. Out of the eight muqarnas, four included the muqarnas of the Nimkar of the Hakim Mosque's eastern entrance, of the Saro-Taghi Mosque's entrance, of the Ilchi Mosque's mihrab, and of the Shishe Mosque's entrance, which lacked the Takht element in the first tier. Meanwhile, the muqarnas of the Nimkar of the Sha'ya Mosque's entrance, of the Jame' Mosque of Isfahan's eastern entrance, of the Hakim Mosque's mihrab, and of the Lonban Mosque featured the element of Takht in the first tier.

There was a discontinuity between elements of muqarnas in half of the plan in the first tier of all muqarnas except for the muqarnas of the Nimkar of the Lonban Mosque.

There was a continuity between elements of muqarnas in half of the plan in the second tier of all muqarnas except for the muqarnas of the nimkar of the Sha'ya Mosque. This indicated that it was only in the second tier of the Nimkar of the Sha'ya Mosque where half of the plan in the second tier featured two adjacent elements not sharing a side with each other.

In the penultimate tier, there was an immediate association between the Tas and the Pabarik in six muqarnas, whereas there was no such association

between the elements in the two muqarnas of the Nimkar of the eastern iwan to the Jame' Mosque and the Nimkar of the entrance to the Shishe Mosque.

The 45° angle was the lowest common multiple between the angles of all muqarnas of the mosques under consideration, except for the muqarnas of the eastern iwan to the Jame' Mosque.

All muqarnas under study featured seven-pointed stars, except for two muqarnas, namely the Nimkar of the eastern iwan of the Jame' Mosque and the Nimkar of the entrance to the Sha'ya Mosque, which featured a star Takht element with over seven points.

In all muqarnas, the arches between the tiers featured 2/3 of the vertical section and 1/3 of the curved section, with the two muqarnas of the eastern iwan of the Jame' mosque and the Ilchi Mosque's mihrab featuring 1/2 of the vertical section and 1/2 of the curved section.

The comparison of two three-tier square Nimkars, i.e., the muqarnas of the Ilchi mihrab and the Nimkar of the Lonban Mosque suggests that the reason why there is no discontinuity between the elements of the half of the plan of the tier in the muqarnas of the Nimkar of the Lonban Mosque is due to the fact that the Toranj's side and the Tanoure Takht were featured to be smaller than the muqarnas of the Ilchi mihrab.

To explain why the two muqarnas that featured no association between the Tas and the Pabarik, and the Tas and the Pabarik in the penultimate tier, one would note that the Nimkar of the eastern iwan to the Jame' Mosque was the only muqarnas among the muqarnas under study that lacked a Tanoure in the penultimate tier. The Nimkar of the entrance to the Shishe Mosque was the only muqarnas in the target population that featured a Madani in another tier except for the last tier, with an association noted between the Madani and the Pabarik rather than the Tas and the Pabarik, despite the formation of the Tanoure in the second tier of this muqarnas.

The 45° angle was not multiple of the Shamse angle in

the Nimkar of the eastern iwan to the Jame' Mosque because the Shamse (Shamse 22) had formed based on a rectangle (due to its specific context), whereas the Shamse in other studied muqarnas had formed based on a square.

The difference of utilizing the Takht with more than seven points between the Nimkar of the eastern iwan to the Jame' Mosque and the Nimkar of the entrance to the Sha'ya Mosque with other muqarnas arises from the specific contexts of these two muqarnas. This means the more the number of Takht points, the more the peripheral axes for the association of the Takht-connected elements such as the Tas and the Pabarik will be, as there should be an adequate surface on the plan for these peripheral elements. The smaller design of the Takht requires the elements sharing a side with it to be designed smaller, also. On the other hand, it would be impossible to design a muqarnas with brick. The highly repetitive composition of the Takht in the Muqarnas plans features the Tanoure with a sum of the five-pointed star Takht, four Pabariks, and a Tas between the Pabariks.

Repetitive features in the studied muqarnas helps plot the plan and facing of the muqarnas. The plan design of the Nimkar of the brick muqarnas with the element of Takht features a circumscribed or inscribed circle of the Shamse or a circumscribed circle of the Toranjs with a radius of half the length (1/4 of the context's length) or half the width of the context (1/2 of the context's width) or half the diameter of the context. Meanwhile, for the side length of Toranj's, half the tier's height can be considered, while for the arches between the tiers, 2/3 of the vertical section and 1/3 of the curved section can be considered, too. Also, the element of the Tanoure Takht is proportionate to the Toranj's side. If a star Takht is less than seven points, the Toranj's side size can be used²⁴. Large angles of the Shaparaks and Pabariks can also be selected to be equivalent to the Toranj's side.

ACKNOWLEDGMENTS

This article wasn't supported by any financial or spiritual sponsors.

CONFLICT OF INTEREST

The authors have no conflicts of interest to declare.

MORAL APPROVAL

The authors commit to observe all the ethical principles of the publication of the scientific work based on the ethical principles of COPE. In case of any violation of the ethical principles, even after the publication of the article, they give the journal the right to delete the article and follow up on the matter.

PARTICIPATION PERCENTAGE

The authors state that they have directly participated in the stages of conducting research and writing the article.

ENDNOTE

1. Stalactite (or stalactite vaulting)
2. Also-Ti an element in muqarnas compositions
3. Element: The components constituting the muqarnas, in terms of shape (two-dimensional geometric shapes) taking convex or concave polygons, and in terms of volume (three-dimensional) which create fill and empty spaces, generally generating muqarnas when they are combined.
4. Takht: In terms of shape, it is often a concave polygon taking the form of a multi-pointed star. It is positioned in a plane parallel to the horizon. Takhts also involve convex types.
5. A slender tower, typically part of a mosque, with a balcony from which a muezzin calls Muslims to prayer
6. Semi-work, in a nimkār, two Toranjs that are located in the far most outer sides of the Karbandi can have two states towards the edge of the iwan
7. Gerehs are decorative Islamic geometric patterns used in architecture and handicraft objects, consisting of angled lines that form an interlaced strapwork
8. Tas: On its two sides stand Tee, or Shaparak, or Toranj, or Pabarik, or Perk. In terms of shape, its sides amount to at least four and can be regular or irregular. In terms of form, two of its sides conjoin the tiers and the rest of the sides are positioned in the horizontal plane.
9. Madani: It is a Tas as high as two tiers or more.
10. Toranj: In terms of shape, it is a convex quadrilateral and resembles a rhombus, but often its two upper sides and two lower sides are equal to each other. The two upper sides of the Toranj are connected to the Shamse. The two lower sides conjoin the tiers.
11. Shaparak: In terms of shape, it features a concave quadrilateral. It often has two small and two large sides. In terms of form, its two smaller sides are positioned in the horizontal plane and the two large sides conjoin the tiers.
- Parak: In terms of shape, it features a convex triangle, but in terms of form, it has a small side in the horizontal plane and the two large sides conjoin the tiers.
12. Moushpa or Tanoure are not separate elements and are referred to as a special case of the togetherness of some elements. Moushpa refers to the Shaparak, Toranj or Parak in the first tier. The Tanoure is a type of togetherness of a number of elements that has the shape of a bowl (in Kase-Sazi). It is created when there is a Tas or Sarseft between the Takht-connected Pabariks by at least half the number of Takht tops
13. Vault-covering elements based on Islamic star geometry
14. Vault-covering elements based on Islamic star geometry
15. altar
16. The Nimkar muqarnas at the entrance to the Ilchi Mosque also fall under this target group; since the registered file of the building describes the destruction and relocation of this entrance back in 1936, so there was some suspicion over the authenticity of this muqarnas based on the original form. In addition, due to the visual damage (painting and photo frame), it was not possible to completely acquire it.
17. Parak: In terms of shape, it features a convex triangle, but in terms of form, it has a small side in the horizontal plane and the two large sides conjoin the tiers.
18. Pabarik: It is similar in shape to Toranj, with the difference being its position. The Pabarik is attached to the Takht, so its two small sides must be positioned in a horizontal plane, but this does not apply to the Toranj.
- Madani: It is a Tas as high as two tiers or more
19. Sarseft: It is a convex triangular shape
20. Tee: It is positioned between two Tases or two Madanis. In terms of shape, it is a very narrow rectangle, and in terms of form, it is a thin blade positioned between two Tases or two Madanis, increasing the depth of the Tas or Madani.
21. This study does not investigate the local reflective symmetry of the elements connected to the Takhts (if only the symmetry is in the elements immediately connected to the Takht).
22. It was not possible to accurately acquire the projected part of the plan, as the plan of this part was plotted based on images of different angles and comparative proportions.
23. By the side length of the Toranj in this table, it is the largest side of the Toranj (the two large sides of the Toranj are equal).
24. Plotting regular polygons by recognizing one side is mentioned on pages 25 to 28 of the Iranian Geometry Book, Application of Geometry in Practice by Bouzjani.

REFERENCES

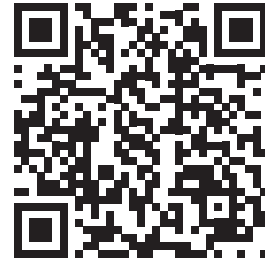
- Agirbas, Asli, Gulnur Yildiz, and Murat Sahin. 2022. "Interrelation between grid systems and star polygons of muqarnas ground projection plans." *Heritage Science* 10(1): 1-26. <https://doi.org/10.1186/s40494-022-00647-z>
- Agirbas, Asli, and Gulnur Yildiz. 2021. "Origin of irregular star polygons in ground projection plans of muqarnas." *Nexus Network Journal* 23: 507-548. <https://doi.org/10.1007/s00004-020-00516-x>
- Alaçam, Sema, and Orkan Zeynel Güzelci. 2016. "Computational interpretations of 2D Muqarnas projections in 3D form finding." 421-430. <https://hdl.handle.net/11413/3005>
- Alaçam, Sema, Orkan Zeynel Güzelci, Ethem Gürer, and Saadet Zeynep Bacınoğlu. 2017. "Reconnoitring computational potentials of the vault-like forms: Thinking aloud on muqarnas tectonics." *International Journal of Architectural Computing* 15(4): 285-303. <https://doi.org/10.1177/1478077117735019>
- Albuzjani, Abolvafa Mohammad Ibn Mohammad. 1997. "Applied Geometry". Translation by Alireza Jazbi. Soroosh. [in Persian]
- Büyükdığan, İ. 2001. "Muqarnas Domes in Edirne". *Architectural Science Review* 44(2): 161-171. <https://doi.org/10.1080/00038628.2001.9697467>
- Dold-Samplonius, Y. 1992. "Practical Arabic Mathematics: Measuring the Muqarnas by al-Kashi". Centaurus: 193-242.
- Fukami, N. 2017. "The Use of Muqarnas in the Transitional Zone of Domes in Egyptian Islamic Architecture". *Orient* 52: 93-119. <https://doi.org/10.5356/orient.52.93>
- Güzelci, O. Z., S. Alaçam, A. K. Şen Bayram, and I. Lacroix. 2021. "Measuring the Entropy of Sinan's Muqarnas Patterns". *Nexus Network Journal* 23: 85-105. <https://doi.org/10.1007/s00004-020-00521-0>
- Güzelci, O., and S. Alaçam. 2019. "A study on measuring complexity in Muqarnas patterns". *Turkish Online Journal of Design Art and Communication* 9(2): 191-201.
- Harmsen, Silvia. 2006. "Algorithmic computer reconstructions of stalactite vaults-muqarnas-in Islamic architecture." PhD diss.
- Kashani, Ghiath Al -Din Jamshid. 2008. "Taq va Azaj". Translation by Alireza Jazbi. Soroosh. [in Persian]
- Kashef, Mohamad. 2017. "Bahri Mamluk muqarnas portals in Egypt: Survey and analysis." *Frontiers of Architectural Research* 6(4): 487-503. <https://doi.org/10.1016/j.foar.2017.09.004>
- KILIÇOĞLU, Selim. 2022. "An Essay on the Classification of Muqarnas: Geometric Foundations Based on Application in Practice." *Türk İslâm Medeniyeti Akademik Araştırmalar Dergisi* 17(34): 331-347. <https://doi.org/10.5281/zenodo.6468626>
- Lorzade, Hossein. 1980. "Revitalization of forgotten arts". Personal print by Khajavi. [in Persian]
- Najib Oghlu, Golru. 2000. "Geometry and decoration in Islamic Architecture, (Topkapi petition)". Translation by Bide Hendi, Mehrdad, Rozaneh, Tehran. [in Persian]
- Özdural, Alpay. 1990. "Gıyaseddin Jemshid El-Kashi and stalactites." *Metujfa*: 31-49. <https://hdl.handle.net/11511/51163>
- Özdural, Alpay. 1991. "Analysis of the geometry of stalactites: Buruciye Medrese". *Metujfa*: 57-71. <https://hdl.handle.net/11511/51150>
- Piran, Hassan. 2018. "Practical training of Muqarnas". Tehran. Sokhanvaran. [in Persian]
- Reiszade, Mahnaz, and Hossein Mofid. 2014. "Revitalization of forgotten arts: Ancient Architecture principles". Tehran: Mola. [in Persian]
- Sakkal, Mamoun. 1988. "An introduction to Muqarnas domes Geometry". *Structural Topology* 1988 núm 14. <http://hdl.handle.net/2099/1042>
- Sharbaf, Asghar. 2006. "Gereh and Karbandi". Tehran: Cultural heritage Organization. [in Persian]
- Takahashi, S. (n.d.). Muqarnas: A Three-Dimensional Decoration of Islam Architecture. Retrieved from <http://www.shiro1000.jp/muqarnas/default.htm>
- Yaghan, Mohammad Ali Jalal. 2001. "The Muqarnas pre-designed erecting units: analysis, definition of the generic set of units, and a system of unit-creation as a new evolutionary step." *Architectural Science Review* 44(3): 297-318. <https://doi.org/10.1080/00038628.2001.9697485>
- Yaghan, Mohammad Ali Jalal. 2010. "The evolution of architectural forms through computer visualisation: muqarnas example." In *Electronic Visualisation and the Arts (EVA 2010)*. BCS Learning & Development: 113-120. <https://www.scienceopen.com/hosted-document?doi=10.14236/ewic/EVA2010.19>
- Zamani, Zahra, Kaveh Fattahi, and Maryam Ekhtiyari. 2022. "Studying Muqarnas Typology and Its Fundamental Components according to Al-Kāshī's Meftāh-ol-Hessāb Treatise." *Golestan-e Honar* 7(2): 16-5. <http://golestane-honar.ir/article-1-328-en.html>. [in Persian]

HOW TO CITE THIS ARTICLE

Rezaei, Elham, Shahryar Nasekhian, and Nima Vali-Beig. 2024. Comparative Study of Muqarnas with Takht and Brick-Facing Elements at Isfahan's Historical Mosques Through a Two-Dimensional Geometry of Muqarnas. *Armanshahr Architecture & Urban Development Journal* 17(48): 65-81.

DOI: 10.22034/AAUD.2024.416947.2828

URL: https://www.armanshahrjournal.com/article_203945.html



COPYRIGHTS

Copyright for this article is retained by the author(s), with publication rights granted to the Armanshahr Architecture & Urban Development Journal. This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution License.

<http://creativecommons.org/licenses/by/4.0/>

